

Exam II: MTH 221, Spring 2016

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QUESTION 1. $((\sqrt{\pi} + 1)/3.14)$. Let D be a subset of R^2 that consists of all points on the x-axis and all points on the y-axis. Then

- (a) D is a subspace of R^2 and $\dim(D) = 1$. (b) D is a subspace of R^2 and $\dim(D) = 2$.
~~(c)~~ D is not a subspace of R^2 because it is not closed under addition (d) D is not a subspace of R^2 because it is not closed under scalar-multiplication.
 (e) D is not a subspace of R^2 because of (c) and (d).

$((\sqrt{\pi} + 2)/3.24)$. Given D is a subspace of R^4 and $\dim(D) = 3$. Then

- (1) Every 3 nonzero points in D form a basis for D . (2) Every 3 independent points in R^4 form a basis for D .
 (3) Every 2 nonzero points in D are independent ~~(4)~~ Every 4 nonzero points in D are dependent.

$((\sqrt{\pi} + 3)/3.34)$. Given $D = \{f(x) \in P_3 \mid f'(1) = 0\}$ is a subspace of P_3 . Then

- (a) $\{x - 1, x^2 - 1\}$ forms a basis for D . (b) $\{x^2 - 2x\}$ forms a basis for D .
 (c) $\{1, x - 1, x^2 - 1\}$ forms a basis for D . ~~(d)~~ $\{1, x^2 - 2x\}$ forms a basis for D .

$((\sqrt{\pi} + 4)/3.44)$. One of the following is not a subspace of $R^{2 \times 2}$.

- (1) $\left\{ \begin{bmatrix} a & a \\ 2b & b \end{bmatrix} \mid a, b \in R \right\}$. ~~(2)~~ $\left\{ \begin{bmatrix} a & a \\ 1 & 3b \end{bmatrix} \mid a, b \in R \right\}$.
 (3) $\left\{ \begin{bmatrix} a & a+b \\ c & 3b \end{bmatrix} \mid a, b, c \in R \right\}$. (4) $\left\{ \begin{bmatrix} -2a & 6a+b \\ 0 & 3c \end{bmatrix} \mid a, b, c \in R \right\}$

$((\sqrt{\pi} + 5)/3.54)$. Given $D = \{f(x) \in P_3 \mid \int_0^1 f(x) dx = 0\}$ is a subspace of P_3 . Then

- ~~(a)~~ $\{2x - 1, 3x^2 - 1\}$ forms a basis for D . (b) $\{2x - 1, 3x^2 - 2x, 6x^2 - 2x - 1\}$ forms a basis for D .
 (c) $\{x - 0.5, x^2 - 2x\}$ forms a basis for D (d) $\{x - 1, x^2 - x\}$ forms a basis for D .

$((\sqrt{\pi} + 6)/3.64)$. Given $D = \text{span}\{(1, 1, 1, 0), (-1, 1, 0, 0), (0, 4, 2, 0)\}$. Then we know that D is a subspace of R^4 . Hence $\dim(D) =$

- (1) 4 (2) 3 ~~(3)~~ 2 (4) 1

$((\sqrt{\pi} + 7)/3.74)$. Given $D = \text{span}\{(1, 1, 1, 0), (-1, 1, 0, 0), (0, 4, 2, 0)\}$. Then we know that D is a subspace of R^4 . One of the following points does not belong (live) in D .

- (a) (0, 6, 3, 0) ~~(b)~~ (4, 0, 1, 0) (c) (2, 0, 1, 0) (d) (0, 0, 0, 0)

$((\sqrt{\pi} + 8)/3.84)$. Given $D = \text{span}\{(1, 1, 1, 0), (-1, 1, 0, 0), (0, 4, 2, 0)\}$. Then we know that D is a subspace of R^3 . One of the following points belongs (lives) in D .

- ~~(1)~~ (5, 1, 3, 0) (2) (2, 0, 2, 0) (3) (2, 0, -2, 0) (4) (0, 3, 1, 0)

$((\sqrt{\pi} + 9)/3.94)$. Given $D = \{(a + 2b, 2a + 4b, 2c) \mid a, b, c \in R\}$. Then we know that D is a subspace of R^4 . One of the following is a basis for D

- (a) $\{(1, 2, 0), (2, 4, 0), (0, 0, 2)\}$ ~~(b)~~ $\{(1, 2, 0), (0, 0, 1)\}$ (c) $\{(1, 2, 0), (0, 0, 1), (0, 2, 0)\}$
 (d) $\{(1, 2, 0)\}$

$((\sqrt{\pi} + 10)/3.104)$. One of the following is a basis for R^3

- (1) $\{(1, 2, 1), (-1, -2, 0), (0, 0, 1)\}$ (2) $\{(1, 2, 0), (0, 2, 1), (-2, -2, 1)\}$
~~(3)~~ $\{(1, 1, 1), (-1, 4, 7), (-2, -2, 9)\}$ (4) $\{(1, 2, 1), (2, 4, 0), (0, 0, 2)\}$

$((\sqrt{\pi} + 11)/3.114)$. Given $D = \{A \in R^{2 \times 2} \mid A^T = -A\}$. Then one of the following is true :

- ~~(a)~~ D is a subspace of $R^{2 \times 2}$. (b) D is not a subspace of $R^{2 \times 2}$ because D is not closed under addition.
 (c) D is not a subspace of $R^{2 \times 2}$ because D is not closed under scalar-multiplication.
 (d) D is not a subspace of $R^{2 \times 2}$ because of (b) and (c)

$(\sqrt{2} + 1)$. Given $D = \{A \in R^{3 \times 3} \mid A \text{ is upper triangular}\}$ is a subspace of $R^{3 \times 3}$. Then $\dim(D) =$

- (1) 1 ~~(2)~~ 6 (3) 2 (4) 3

$(\sqrt{2} + 2)$. Given Let $A = \begin{bmatrix} 4 & 0 & a_1 \\ 8 & 8 & a_2 \\ 12 & 8 & a_3 \end{bmatrix}$ such that $|A| = 20$. Consider the system of linear equations $AX = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}$. Then the value of x_3 is

- (a) 0 (b) 5 ~~(c)~~ 8 (d) 4 (e) More information is needed.

$(\sqrt{2} + 3)$. Given Let $A = \begin{bmatrix} 4 & 0 & a_1 \\ 8 & 8 & a_2 \\ 12 & 8 & a_3 \end{bmatrix}$ such that $|A| = 20$. Consider the system of linear equations $AX = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$. Then the value of x_1 is

- ~~(1)~~ 0 (2) $\frac{1}{2}$ (3) 2 (4) $\frac{1}{4}$ (5) 4

$(\sqrt{2} + 4)$. One of the following is a basis for P_3

- ~~(a)~~ $\{x^2, x^2 + x, x^2 + 1\}$ (b) $\{1, x^2 + x, 2x^2 + 2x + 3\}$ (c) $\{1, x^2 + 1, x^2 + 5\}$ (d) $\{x^2 + 1, x + 1, x^2 + 2x + 3\}$.

$(\sqrt{2} + 5)$. Given A is row-equivalent to $B = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 0 & 1 & 1 \\ 0 & 0 & -1 & 0 \\ -2 & 0 & 0 & 2 \end{bmatrix}$. Then $N(A)$ is

- (1) $\text{span}\{(1, 1, 0, -1)\}$ (2) $\text{span}\{(1, 0, 0, -1)\}$ ~~(3)~~ $\text{span}\{(1, 0, 0, 1), (0, 1, 0, 0)\}$ (4) $\text{span}\{(1, 0, 0, 0), (0, 1, 0, 0)\}$

$(\sqrt{2} + 6)$. Let A be a 3×3 matrix such that $C_A(\alpha) = (\alpha - 2)(\alpha - a)^2$ for some real number a . Given $\text{trace}(A) = 12$. Then $|A| =$

- (a) 20 (b) 10 (c) 24 ~~(d)~~ 50 (e) 12

$(\sqrt{2} + 7)$. Given A is a 3×3 such that $C_A(\alpha) = (\alpha - 3)(\alpha - 2)(\alpha - 4)$. Then one of the following is true:

- ~~(1)~~ A is similar to $D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ (2) $|A| = 9$ (3) $\text{trace}(A + I) = 10$ (4) $|A^T| = \frac{1}{24}$.

$(\sqrt{2} + 8)$. Let A be a 3×3 matrix such that $C_A(\alpha) = (\alpha - 2)(\alpha - 1)^2$, $E_1 = \text{span}\{(1, 1, 0), (0, 1, 0)\}$ and $E_2 = \text{span}\{(-1, -1, 3)\}$. Then we know that there is an invertible matrix Q such that $Q^{-1}AQ = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. One possibility for Q is

- (a) $Q = \begin{bmatrix} 1 & -1 & 2 \\ 1 & -1 & 2 \\ 0 & 3 & 0 \end{bmatrix}$ ~~(b)~~ $Q = \begin{bmatrix} 1 & -2 & 0 \\ 1 & -2 & 1 \\ 0 & 6 & 0 \end{bmatrix}$ (c) $Q = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & -1 \\ 0 & 0 & 3 \end{bmatrix}$ (d) $Q = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 1 & 1 \\ 3 & 0 & 0 \end{bmatrix}$

$(\sqrt{2} + 9)$. Let A be a 3×3 matrix such that $C_A(\alpha) = \alpha(\alpha - 2)(\alpha - 1)$. Then $|A^2 + I| =$

- (1) 1 (2) 36 ~~(3)~~ 10 (4) 6

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